

HEAVY MESON HYPERFINE SPLITTING: A COMPLETE $1/m_Q$ CALCULATION.¹

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ABSTRACT

The hyperfine splittings $\Delta_D = (m_{D_s^*} - m_{D_s}) - (m_{D^{*+}} - m_{D^+})$ and $\Delta_B = (m_{B_s^*} - m_{B_s}) - (m_{B^{*0}} - m_{B^0})$ are analyzed in the framework of an effective lagrangian possessing chiral, heavy flavour and spin symmetries, explicitly broken by a complete set of first order terms. Among these terms, those responsible for the difference between the couplings $g_{P^*P^*\pi}$ and $g_{P^*P\pi}$ are evaluated in the QCD sum rules approach. Their contribution to Δ_D and to Δ_B appears to quantitatively balance previously estimated chiral effects in agreement with the experimental data, solving a suspected puzzle for heavy quark theory.

1 Introduction

The spectroscopy of heavy mesons is among the simplest framework where the ideas and the methods of heavy quark expansion can be quantitatively tested. Recently, attention has been focused on the combinations [1, 2, 3, 4]:

$$\Delta_D = (m_{D_s^*} - m_{D_s}) - (m_{D^{*+}} - m_{D^+}) \quad (1.1)$$

$$\Delta_B = (m_{B_s^*} - m_{B_s}) - (m_{B^{*0}} - m_{B^0}) \quad (1.2)$$

which are measured to be [5]:

$$\Delta_D \simeq 1.0 \pm 1.8 \text{ MeV} \quad (1.3)$$

$$\Delta_B \simeq 1.0 \pm 2.7 \text{ MeV} \quad (1.4)$$

The above hyperfine splitting is free from electromagnetic corrections and it vanishes separately in the $SU(3)$ chiral limit and in the heavy quark limit. In the combined chiral and heavy quark expansion, the leading contribution is of order m_s/m_Q and one would expect the relation [1]:

$$\Delta_B = \frac{m_c}{m_b} \Delta_D \quad (1.5)$$

In the so called heavy meson effective theory [6], which combines the heavy quark expansion and the chiral symmetry, there is only one lowest order operator contributing to $\Delta_{D,B}$:

$$\lambda_2 \mathcal{O}_2 = \frac{\lambda_2}{8} \text{Tr}[\bar{H}_a^i \sigma_{\mu\nu} H_j^b \sigma^{\mu\nu}] (m_Q^{-1})_i^j \frac{(m_\xi)_b^a}{\Lambda_{CSB}} \quad (1.6)$$

where i, j are heavy flavour indices and a, b light flavour indices. The 4×4 Dirac matrix H_a^i describes the spin doublet P, P^* , with P heavy meson composed by the heavy quark Q_i and the light antiquark \bar{q}_a . The matrix m_ξ is

$$m_\xi = (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger) \quad (1.7)$$

Here m_q is the light quarks mass matrix and $\xi = \exp(iM/f)$, where M is the pseudoscalar 3×3 matrix and f the pseudoscalar decay constant (we take $f = 132 \text{ MeV}$).

By taking $m_s/\Lambda_{CSB} \simeq 0.15$ and by taking $\lambda_2 \simeq \Lambda_{QCD}^2 \simeq 0.1 \text{ GeV}^2$ one would estimate:

$$\Delta_D^{(2)} \simeq 20 \text{ MeV} \quad (1.8)$$

$$\Delta_B^{(2)} \simeq 6 \text{ MeV} \quad (1.9)$$

Given the present experimental accuracy, the above estimate is barely acceptable, as an order of magnitude, for Δ_B , while it clearly fails to reproduce the data for Δ_D . If the contribution from \mathcal{O}_2 were the only one responsible for the hyperfine splittings, agreement with the data clearly would require a rather small value for λ_2 .

In chiral perturbation theory, an independent contribution arises from one-loop corrections to the heavy meson self energies [3], evaluated from an initial lagrangian containing,

at the lowest order, both chiral breaking and spin breaking terms. The loop corrections in turn depend on an arbitrary renormalization point μ^2 (e.g. the t'Hooft mass of dimensional regularization). This dependence is cancelled by the μ^2 dependence of the counterterm $\lambda_2(\mu^2)\mathcal{O}_2$, as it should happen for any physical result. A commonly accepted point of view is that the overall effect of adding the counterterm consists in replacing μ^2 in the loop corrections with the physical scale relevant to the problem at hand, Λ_{CSB}^2 . Possible finite terms in the counterterm are supposed to be small compared to the large chiral logarithms. With this philosophy in mind, two classes of such corrections has been estimated in ref. [2] and they give (for the values of the parameters given by these authors):

$$\Delta_D^0 \simeq +30 \text{ MeV}, \quad \Delta_D^1 \simeq +65 \text{ MeV}, \quad (1.10)$$

$$\Delta_B^0 \simeq +10 \text{ MeV}, \quad \Delta_B^1 \simeq +22 \text{ MeV}, \quad (1.11)$$

Here Δ^0 represents the contribution of the chiral logarithm and Δ^1 is a non analytic contribution, of order $m_s^{3/2}$. This provides a rather uncomfortable situation since, to account for the observed data, one should require an accurate and innatural cancellation between $(\Delta^0 + \Delta^1)$ and the finite terms from $\Delta^{(2)}$, contrary to the usual expectation.

The chiral computation giving $\Delta^0 + \Delta^1$ is however incomplete [4], because it does not include the spin breaking effect due to the difference between the $P^*P^*\pi$ and the $P^*P\pi$ couplings ($P = D, B$), defined by the relations:

$$\langle \pi^-(q) | P^o(q_2) | P^{*-}(q_1, \epsilon) \rangle = 2 g_{P^*P\pi} \frac{m_P}{f_\pi} \epsilon^\mu \cdot q_\mu \quad (1.12)$$

$$\langle \pi(q) | P^*(q_2, \epsilon_2) | P^*(q_1, \epsilon_1) \rangle = -i \frac{2}{f_\pi} g_{P^*P^*\pi} \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_2^\nu q^\alpha q_1^\beta \quad (1.13)$$

The scaling law $2g_{P^*P\pi}m_P/f_\pi$ for the strong $D^*D\pi$ coupling constant was first proposed in [7, 8]. The splitting between the couplings (1.12) and (1.13) is of order $1/m_Q$, and therefore has to be taken into account in the chiral computation, to work consistently at the desired order.

In the present paper we will provide an estimate of $g_{P^*P^*\pi} - g_{P^*P\pi}$ based on a QCD sum rule, and, by including this additional spin breaking effect, we will complete the evaluation of $\Delta_{D,B}$ coming from the chiral loops.

2 The Hyperfine Splitting

To better clarify the importance of $g_{P^*P^*\pi} - g_{P^*P\pi}$ for the problem at hand, we remind that the effective lagrangian for heavy mesons and light pseudoscalars, at first order in m_Q^{-1} and in the light quark masses m_q reads:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}^q + \mathcal{L}^Q \quad (2.1)$$

Here \mathcal{L}_0 represents the chiral, heavy flavour and spin symmetric term:

$$\begin{aligned}\mathcal{L} = & -iTr[\bar{H}_a^i v_\mu \partial^\mu H_i^a] + \frac{f^2}{8}Tr[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma] \\ & + \frac{i}{2}Tr[\bar{H}_a^i H_i^b] v^\mu (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_b^a \\ & + \frac{i}{2}gTr[\bar{H}_a^i H_i^b \gamma_\mu \gamma_5](\mathcal{A}^\mu)_b^a\end{aligned}\quad (2.2)$$

where $\Sigma = \xi^2$ and:

$$\mathcal{A}_\mu = \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \quad (2.3)$$

From the last term in eq. (2.2), one obtains the $P^*P\pi$ and $P^*P^*\pi$ couplings defined in eqs. (1.12) and (1.13), in the limit $m_P \rightarrow \infty$:

$$g_{P^*P\pi} = g_{P^*P^*\pi} = g \quad (2.4)$$

The leading chiral breaking corrections are given by:

$$\begin{aligned}\mathcal{L}^q = & \lambda_0 Tr[m_q \Sigma + \Sigma^\dagger m_q] \\ & + \lambda_1 Tr[\bar{H}_a^i H_i^b](m_\xi)_b^a \\ & + \lambda'_1 Tr[\bar{H}_a^i H_i^a](m_\xi)_a^a\end{aligned}\quad (2.5)$$

The second term in eq. (2.5) is responsible for the mass splitting between strange and non-strange heavy mesons:

$$\Delta_s = 2\lambda_1 m_s \quad (2.6)$$

One has approximately $\Delta_s \simeq 100 \text{ MeV}$, $\lambda_1 \simeq 0.33$.

The third term, listed for completeness, gives an equal contribution to each heavy meson mass, it does not affect the hyperfine splitting, and it does not play any role in our analysis.

Finally the terms of order $1/m_Q$, breaking either the heavy flavour or the spin symmetries, are given by:

$$\begin{aligned}\mathcal{L}^Q = & -\frac{\lambda}{8}Tr[\bar{H}_a^i \sigma_{\mu\nu} H_j^a \sigma^{\mu\nu}](m_Q^{-1})_i^j \\ & + \frac{ig(a+b)}{2}Tr[\bar{H}_a^i H_j^b \gamma_\mu \gamma_5](m_Q^{-1})_i^j (\mathcal{A}^\mu)_b^a \\ & + \frac{ig(a-b)}{2}Tr[\bar{H}_a^i \gamma_\mu \gamma_5 H_j^b](m_Q^{-1})_i^j (\mathcal{A}^\mu)_b^a\end{aligned}\quad (2.7)$$

The first term in eq. (2.7) is responsible for the splitting Δ between the 1^- and 0^- heavy meson masses:

$$\Delta = \frac{2\lambda}{m_Q} \quad (2.8)$$

For the B, B^* system $\Delta \simeq 46 \text{ MeV}$, whereas for D, D^* Δ is approximately 141 MeV , so that one has:

$$\lambda \simeq 0.10 - 0.11 \text{ GeV}^2 \quad (2.9)$$

The second term in (2.7) breaks only the heavy flavour symmetry, making the $B^* B^{(*)} \pi$ and $D^* D^{(*)} \pi$ couplings different. The third term breaks also the spin symmetry and contributes differently to the $P^* P \pi$ and to the $P^* P^* \pi$ couplings. This is precisely the effect relevant to the hyperfine splitting. To this order in $1/m_Q$ one has:

$$g_{P^* P^* \pi} = g \left(1 + \frac{a}{m_Q} \right) \quad g_{P^* P \pi} = g \left(1 + \frac{b}{m_Q} \right) \quad (2.10)$$

and

$$\Delta_g \equiv g_{P^* P^* \pi} - g_{P^* P \pi} = g \frac{a - b}{m_Q} \quad (2.11)$$

The chiral and spin symmetry breaking parameters relevant to the hyperfine splitting are the light pseudoscalar masses m_π , m_K and m_η , Δ_s , Δ and Δ_g . In terms of these quantities, one finds [3, 2, 4]:

$$\begin{aligned} \Delta_P &= \frac{g^2 \Delta}{16\pi^2 f^2} \left[4m_K^2 \ln\left(\frac{\Lambda_{CSB}^2}{m_K^2}\right) + 2m_\eta^2 \ln\left(\frac{\Lambda_{CSB}^2}{m_\eta^2}\right) - 6m_\pi^2 \ln\left(\frac{\Lambda_{CSB}^2}{m_\pi^2}\right) \right] \\ &+ \frac{g^2 \Delta}{16\pi^2 f^2} [24\pi m_K \Delta_s] \\ &- \frac{g^2}{6\pi f^2} \frac{\Delta_g}{g} \left(m_K^3 + \frac{1}{2} m_\eta^3 - \frac{3}{2} m_\pi^3 \right) \end{aligned} \quad (2.12)$$

The dependence upon the heavy flavour $P = D, B$ is contained in the parameters Δ and Δ_g .

The first term in eq. (2.12) is the so called chiral logarithm [3]. In the ideal situation with pseudoscalar masses much smaller than Λ_{CSB} , it would represent the dominant contribution to Δ_P . For the case of D and B mesons the corresponding values have been listed in eqs. (1.10) and (1.11) as Δ_D^0 and Δ_B^0 , respectively. There a value $g^2 = 0.5$ has been used.

The second term in eq. (2.12) represents a non analytic contribution of order $m_s^{3/2}$ [2], which, although formally suppressed with respect to the leading one, is numerically more important, because of the large coefficient 24π . It is given by Δ_D^1 and Δ_B^1 in eqs. (1.10) and (1.11).

Finally, the last term in eq. (2.12) [4] is also of order $m_s^{3/2}$. It can be numerically important as soon as Δ_g/g is of order 10% and, if equipped with the right sign, it can cause a substantial cancellations of the previous two contributions.

3 QCD Sum Rules for $g_{P^*P\pi}$ and $g_{P^*P^*\pi}$

The coupling $g_{P^*P\pi}$ has already been calculated in [9] by means of QCD sum rules and here we proceed to a similar computation concerning the coupling $g_{P^*P^*\pi}$. We start from the correlator:

$$A_{\mu\nu}(q_1, q) = i \int dx \langle \pi(q) | T(V_\mu(x) V_\nu^\dagger(0)) | 0 \rangle e^{-iq_1 x} = A(q_1^2, q_2^2, q^2) \epsilon_{\mu\nu\alpha\beta} q^\alpha q_1^\beta + \dots \quad (3.1)$$

where $V_\mu = \bar{u}\gamma_\mu Q$ is the interpolating vector current for the P^* meson.

We compute the scalar function A in the soft pion limit $q \rightarrow 0$. This implies $q_1 = q_2$ forcing to use a single Borel transformation, and it is the origin of the so called parasitic terms [9]. The correlator in (3.1) can be calculated by an Operator Product Expansion: we keep all the operators with dimension up to five, arising from the expansion of the current $V_\mu(x)$ at the third order in power of x , and the heavy quark propagator to the second order. The result is:

$$\begin{aligned} A(q_1^2, q_1^2, 0) &= \frac{f_\pi}{q_1^2 - m_b^2} + \frac{1}{(q_1^2 - m_b^2)^2} \left[\frac{2 \langle \bar{u}u \rangle m_b}{3f_\pi} + \frac{8f_\pi m_1^2}{9} \right] \\ &+ \frac{1}{(q_1^2 - m_b^2)^3} \left[-\frac{10m_b^2 f_\pi m_1^2}{9} + \frac{m_0^2 \langle \bar{u}u \rangle m_b}{3f_\pi} \right] + \\ &- \frac{1}{(q_1^2 - m_b^2)^4} \frac{m_0^2 \langle \bar{u}u \rangle m_b^3}{f_\pi} \end{aligned} \quad (3.2)$$

In eqs.(3.2) $\langle \bar{u}u \rangle$ is the quark condensate ($\langle \bar{u}u \rangle = -(240 \text{ MeV})^3$), m_0 and m_1 are defined by the equations

$$\langle \bar{u}g_s \sigma \cdot Gu \rangle = m_0^2 \langle \bar{u}u \rangle \quad (3.3)$$

$$\langle \pi(q) | \bar{u} D^2 \gamma_\mu \gamma_5 d | 0 \rangle = -i f_\pi m_1^2 q_\mu \quad (3.4)$$

and their numerical values are: $m_0^2 = 0.8 \text{ GeV}^2$, $m_1^2 = 0.2 \text{ GeV}^2$ [10, 11].

Proceeding in a standard way, we now compute the hadronic side of the sum rule. We can write down for $A(q_1^2, q_2^2, 0)$ the following dispersion relation:

$$A(q_1^2, q_2^2, 0) = \frac{1}{\pi^2} \int ds ds' \frac{\rho(s, s')}{(s - q_1^2)(s' - q_2^2)}. \quad (3.5)$$

It should be observed that we have not written down in (3.5) subtraction terms because, as proven in [12], only a subtraction polynomial $P_3(q_1^2, q_2^2)$ could be present in (3.5), but it would vanish after the Borel transform.

We divide the integration region in three parts [9]. The first region (I) is the square given by $m_b^2 \leq s, s' \leq s_0$ and it contains only the B^* pole, whose contribution is

$$A_I(q_1^2, q_2^2, 0) = \frac{-2g_{B^*B^*\pi} f_{B^*}^2 m_{B^*}^2}{f_\pi (q_1^2 - m_{B^*}^2)(q_2^2 - m_{B^*}^2)} \quad (3.6)$$

where f_{B^*} is defined by

$$\langle 0|V_\mu(0)|B^*(\epsilon, p) \rangle = \epsilon_\mu f_{B^*} m_{B^*} \quad (3.7)$$

The second (II) integration region is defined as follows: $m_b^2 \leq s \leq s_0$ and $s' > s_0$ or $m_b^2 \leq s' \leq s_0$ and $s > s_0$. Here we obtain a contribution coupling the vector current V_μ to the pion and the B^* . Introducing the form factor V as

$$\langle \pi(q)|V_\mu|B^*(q_1, \epsilon) \rangle = V(q_2^2) \epsilon_{\mu\nu\alpha\beta} \epsilon^\nu q_1^\alpha q^\beta \quad (3.8)$$

where $q_2 = q_1 - q$, we get

$$A_{II}(q_1^2, q_2^2, 0) = f_{B^*} m_{B^*} \left(\frac{V(q_2^2)}{q_1^2 - m_{B^*}^2} + \frac{V(q_1^2)}{q_2^2 - m_{B^*}^2} \right) \quad (3.9)$$

In the previous formula one does not have to include the B^* pole contribution to $V(q^2)$, being already taken into account in A_I . We assume that, taken away B^* , a single higher resonance of mass m' contribute to V

$$V_{res}(q^2) = \frac{k}{q^2 - m'^2} \quad (3.10)$$

where k is an unknown constant.

The third region is defined by $s, s' > s_0$, and under the assumption of duality it should coincide with the asymptotic limit $q_1^2 = q_2^2 \rightarrow -\infty$ in (3.2). One gets:

$$A_{III}(q_1^2, q_1^2, 0) = \frac{f_\pi}{q_1^2 - s_0} \quad (3.11)$$

The hadronic side of the sum rule is the sum of the contributions from the three regions:

$$A_I(q_1^2, q_1^2, 0) = \frac{-2g_{B^*B^*\pi} f_{B^*}^2 m_{B^*}^2}{f_\pi (q_1^2 - m_{B^*}^2)(q_2^2 - m_{B^*}^2)} + \frac{k'}{(q_1^2 - m_{B^*}^2)(q_1^2 - m'^2)} + \frac{f_\pi}{q_1^2 - s_0} \quad (3.12)$$

We have put $q_1 = q_2$ and $k' = m_{B^*} f_{B^*} k$.

Equating now the hadronic and the QCD sides of the sum rule, respectively given by eq. (3.12) and (3.2), and taking the Borel transform with parameter M^2 we find:

$$\begin{aligned} \frac{2g_{B^*B^*\pi} m_{B^*}^2 f_{B^*}^2}{f_\pi M^2} + k' + \exp(-\delta/M^2)(f_\pi - k') &= \\ &= \exp(\Omega/M^2) \left[f_\pi - \frac{1}{M^2} \left(\frac{2 \langle \bar{u}u \rangle m_b}{3f_\pi} + \frac{8f_\pi m_1^2}{9} \right) \right. \\ &\quad \left. - \frac{1}{M^4} \left(\frac{5m_b^2 f_\pi m_1^2}{9} - \frac{m_0^2 \langle \bar{u}u \rangle m_b}{6f_\pi} \right) + \frac{1}{M^6} \frac{m_0^2 \langle \bar{u}u \rangle m_b^3}{6f_\pi} \right] \\ &= \exp \Omega/M^2 S(M^2) \end{aligned} \quad (3.13)$$

In the previous formula we have put $m'^2 \simeq s_0$ and we have introduced the parameters $\delta = s_0 - m_{B^*}^2$ and $\Omega = m_{B^*}^2 - m_b^2$.

Differentiating (3.13) respect to the variable $1/M^2$ and combining the first and second derivatives in order to eliminate the unknown parameter k' , we obtain the following sum rule:

$$g_{B^*B\pi} = \frac{f_\pi}{2m_{B^*}^2 f_{B^*}^2} \frac{\exp \Omega/M^2}{\delta} \left[\Omega(\Omega + \delta) S(M^2) + (2\Omega + \delta) \partial_{1/M^2} S(M^2) + \partial_{1/M^2}^2 S(M^2) \right] \quad (3.14)$$

To eliminate the parameter k' one could also combine the first derivative with the original sum rule (3.13): we have checked that the two procedures give the same numerical results. We have used the second derivative to make an easy comparison with the sum rule for $g_{B^*B\pi}$ [9]:

$$g_{B^*B\pi} = \frac{4f_\pi m_b^2 m_{B^*}}{2m_B^3 f_B f_{B^*} (3m_{B^*}^2 + m_B^2)} \frac{\exp(\Omega'/M^2)}{(\delta' - \delta' \Delta_{B^*B}/M^2 - \Delta_{B^*B})} \times \left[\Omega'(\Omega' + \delta') S'(M^2) + (2\Omega' + \delta') \partial_{1/M^2} S'(M^2) + \partial_{1/M^2}^2 S'(M^2) \right] \quad (3.15)$$

where $\Delta_{B^*B} = m_{B^*}^2 - m_B^2$, $\delta' = s_0 - m_B^2 = \delta + \Delta_{B^*B}$, $\Omega' = m_B^2 - m_b^2 = \Omega - \Delta_{B^*B}$ and

$$\begin{aligned} S'(M^2) &= f_\pi - \frac{\langle \bar{u}u \rangle}{3m_b f_\pi} \\ &+ \frac{1}{M^2} \left[-\frac{2\langle \bar{u}u \rangle m_b}{3f_\pi} + \frac{10f_\pi m_1^2}{9} + \frac{m_0^2 \langle \bar{u}u \rangle}{3m_b f_\pi} \right] \\ &+ \frac{1}{2M^4} \left[-\frac{10m_b^2 f_\pi m_1^2}{9} + \frac{m_0^2 \langle \bar{u}u \rangle m_b}{6f_\pi} \right] + \\ &+ \frac{m_0^2 \langle \bar{u}u \rangle m_b^3}{6f_\pi M^6} \end{aligned} \quad (3.16)$$

Eq. (3.15) differs slightly from the one given in [9], since it keeps track of the mass difference Δ_{B^*B} . The sum rules (3.14) and (3.15) have to be analyzed in the duality region, i.e. the region in M^2 where there exists a hierarchy among the different contributions of higher dimension operators (this fixes the lower bound for M^2); moreover we impose that the contribution of the parasitic term does not exceed that of the resonance term, which fixes the upper bound in M^2 . In this way we obtain for the B M^2 in the range $20-40 \text{ GeV}^2$ (for $s_0 = 33-36 \text{ GeV}^2$), and for the D $M^2 = 4-7 \text{ GeV}^2$ (for $s_0 = 6-8 \text{ GeV}^2$). Using $m_b = 4.6 \text{ GeV}$ and $m_c = 1.34 \text{ GeV}$ one gets:

$$\begin{aligned} f_{B^*}^2 g_{B^*B\pi} &= 0.0094 \pm 0.0018 \text{ GeV}^2 \\ f_{D^*}^2 g_{D^*D\pi} &= 0.017 \pm 0.004 \text{ GeV}^2; \end{aligned} \quad (3.17)$$

and for the $g_{P^*P\pi}$ coupling

$$\begin{aligned} f_B f_{B^*} g_{B^*B\pi} &= 0.0074 \pm 0.0014 \text{ GeV}^2 \\ f_D f_{D^*} g_{D^*D\pi} &= 0.0112 \pm 0.0030 \text{ GeV}^2; \end{aligned} \quad (3.18)$$

Once multiplied by $2m_P/f_\pi$ the figures in (3.18) agree with those given in [9].

A recent calculation of the quantity reported in Eq. (3.18) has been given in Ref. [13]. When expressed in our units their results are as follows: $f_B f_{B^*} g_{B^*B\pi} = 0.0079 \pm 0.0007 \text{ GeV}^2$ and $f_D f_{D^*} g_{D^*D\pi} = 0.018 \pm 0.002 \text{ GeV}^2$. The result for the B is only slightly larger than our outcome Eq. (3.18), whereas the result for the D is significantly ($\simeq 60\%$) larger. The origin of the discrepancy is in the different range of values for the Borel parameter M^2 , that, in the case of Ref. [13], are generally smaller. A possible origin of this difference is the fact that, while in this paper we use QCD sum rules in the soft pion limit, in [13] light cone sum rules are adopted, which results in an expansion in operators of increasing twist instead of increasing dimension. In particular we have included a dimension 5 contribution which is proportional to the $m_0^2 < \bar{u}u >$ condensate. This term has no counterpart in [13]; since it has to be kept small, its inclusion in [13] might result in a more stringent constraint on the hierarchy among the different contributions of the Operator Product Expansion, and, therefore, in a more stringent lower limit on M^2 .

We now expand the sum rules (3.14) and (3.15) in the parameter $1/m_Q$, keeping the leading term and the first order corrections. The leading term is the one surviving in the limit $m_b \rightarrow \infty$, and has already been calculated in [9] for $g_{P^*P\pi}$.

To extract the $1/m_Q$ corrections we introduce the following parameters, finite in the large mass limit:

$$E = \frac{M^2}{2m_b} ; y_0 = \frac{s_0 - m_b^2}{2m_b} ; \omega = m_B - m_b \quad (3.19)$$

and the $1/m_Q$ corrections to the leptonic decay constants, $g_{P^*P\pi}$ and $g_{P^*P^*\pi}$

$$f_M = \frac{\hat{F}}{\sqrt{m_Q}} \left(1 + \frac{A}{m_Q} \right) \quad f_{M^*} = \frac{\hat{F}}{\sqrt{m_Q}} \left(1 + \frac{A'}{m_Q} \right) \quad (3.20)$$

The coefficients A and A' have been computed in [14] [15], but only for the B mesons. The corrections found are large and suffering of large uncertainties: there are significant numerical differences between [14] and [15].

In the limit $m_Q \rightarrow \infty$ the right hand sides of (3.14) and (3.15) coincide (notice that $S = S'$ in this limit), confirming the result anticipated in (2.4) and giving:

$$g\hat{F}^2 = \frac{f_\pi \exp(\omega/E)}{(y_0 - \omega)} \left[y_0 S_0(E) + (y_0 + \omega) \partial_{1/E} S_0(E) + \partial_{1/E}^2 S_0(E) \right] \quad (3.21)$$

where

$$S_0(E) = f_\pi - \frac{< \bar{u}u >}{3f_\pi E} - \frac{5f_\pi m_1^2}{36E^2} + \frac{m_0^2 < \bar{u}u >}{48E^3 f_\pi} \quad (3.22)$$

The duality region extends for $E = 4 - 6 \text{ GeV}$ and $y_0 = 1.1 - 1.3 \text{ GeV}$. Numerically one obtains:

$$\hat{F}^2 g = 0.040 \pm 0.005 \text{ GeV}^3 \quad (3.23)$$

While this result agrees within the error with that given in [9], the central value reported here is 15 % larger due to a slightly different choices of the phenomenological parameters.

We can then write the $1/m_Q$ expansion for the function S and S' :

$$S(E) = S_0(E) + \frac{S_1(E)}{m_Q} \quad S'(E) = S_0(E) + \frac{S'_1(E)}{m_Q} \quad (3.24)$$

where

$$\begin{aligned} S_1(E) &= -\frac{4f_\pi m_1^2}{9E} + \frac{m_0^2 \langle \bar{u}u \rangle}{24E^2 f_\pi} \\ S'_1(E) &= -\frac{\langle \bar{u}u \rangle}{3f_\pi} + \frac{5f_\pi m_1^2}{9E} + \frac{m_0^2 \langle \bar{u}u \rangle}{48E^2 f_\pi} \end{aligned} \quad (3.25)$$

From (3.14) and (3.15), one gets the following sum rules for the parameters a and b :

$$\begin{aligned} a &= \frac{f_\pi \exp \omega/E}{2g\hat{F}^2(y_0 - \omega)} \left[(\omega^2 + 4\lambda)(y_0 S_0(E) + \partial_{1/E} S_0(E)) + \right. \\ &+ 2y_0 \omega S_1(E) + 2(y_0 + \omega) \partial_{1/E} S_1(E) + 2\partial_{1/E}^2 S_1(E) \left. \right] + \\ &+ \frac{\omega^2 + 4\lambda}{2E} + \frac{\omega^2 + 4\lambda}{2(y_0 - \omega)} - 2(A' + \omega) \end{aligned} \quad (3.26)$$

$$\begin{aligned} b &= \frac{f_\pi \exp \omega/E}{2g\hat{F}^2(y_0 - \omega)} \left[\omega^2(y_0 S_0(E) + \partial_{1/E} S_0(E)) + \right. \\ &+ 2y_0 \omega S'_1(E) + 2(y_0 + \omega) \partial_{1/E} S'_1(E) + 2\partial_{1/E}^2 S'_1(E) \left. \right] + \\ &+ \frac{\omega^2 + 4\lambda}{2E} + \frac{\omega^2 + 4\lambda}{2(y_0 - \omega)} - (A + A' + 4\omega) \end{aligned} \quad (3.27)$$

where λ has been given in (2.9). From the previous sum rules one gets:

$$a + 2A' = -0.15 \pm 0.20 \text{ GeV} \quad b + A' + A = -1.15 \pm 0.20 \text{ GeV} \quad (3.28)$$

and

$$a - b + (A' - A) = 0.99 \pm 0.02 \text{ GeV} \quad (3.29)$$

Notice that the difference has a quite smaller uncertainty, due to a partial cancellation of terms depending on the threshold.

Neglecting radiative corrections, A and A' are given by [14, 15]:

$$A = -\omega + \frac{G_K}{2} + 3G_\Sigma \quad A' = -\frac{\omega}{3} + \frac{G_K}{2} - G_\Sigma \quad (3.30)$$

Notice that the splitting of the couplings depends on the quantity $a - b$ that contains only the difference $A' - A$ given by:

$$A' - A = \frac{2}{3}\omega - 4G_\Sigma \quad (3.31)$$

There is disagreement in the literature on the values of the parameter G_Σ : at the b quark mass scale from ref.[14] one gets $G_\Sigma = (0.042 \pm 0.034 \pm 0.023 \pm 0.030) \text{ GeV}$, while in ref. [15] the central value $G_\Sigma \simeq -(0.052) \text{ GeV}$ is quoted. In view of this discrepancy, to provide an estimate of the difference (3.31), we will approximate $A' - A \approx 2/3\omega \approx 0.4 \text{ GeV}$, obtaining

$$a - b \approx 0.6 \text{ GeV} \quad (3.32)$$

4 Discussion and conclusions

From (2.11), (3.32) and from the formula (2.12) of the hyperfine mass splitting we obtain:

$$\Delta_B \approx g^2(27.3 + 61.4 - 75.8) \text{ MeV} = 12.9g^2 \text{ MeV} \quad (4.1)$$

Notice that we have used in eq. (2.12) $f = f_\pi = 132 \text{ MeV}$ for all the light pseudoscalar mesons of the octet. This is suggested by the sum rule for g which shows that g/f is flavour independent. In eq. (4.1) we have detailed the contributions Δ^0 , Δ^1 and the one from Δ_g/g respectively. We have also taken $\Lambda_{CSB} = 1 \text{ GeV}$. It is evident that there is a large cancellation among the last term and the other ones. In order to be more quantitative we have to fix the value of g , which, on the basis of our result (3.23), depends on the value of \hat{F} . In Ref. [9] the range of values $g \simeq 0.2 - 0.4$ was found; therefore, putting $g^2 = 0.1$, we would obtain

$$\Delta_B \simeq 1.3 \text{ MeV} \quad (4.2)$$

The application of our results to the charm case is more doubtful, in view of the large values of the $1/m_c$ correction $(a - b)/m_c$. By scaling the result (4.2) to the charm case, one obtains

$$\Delta_D = \frac{m_b}{m_c} \Delta_B \simeq 4.4 \text{ MeV} \quad (4.3)$$

In conclusion, our estimate of $g_{P^*P^*\pi} - g_{P^*P\pi}$ allows to include a previously neglected term in the loop induced contribution to the hyperfine splitting. Although our estimate is affected by an uncertainty in the value of G_Σ , nevertheless this new term tends to cause a substantial cancellation and to reconcile the chiral calculation with the experimental data.

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